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MODEL MULTILINEAL PATTERN FORMATION: A COMPUTER EXPERIMENT

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Abstract. Multilineal pattern formation in the square lattice is studied. It turns out that lognormality both in space (size) and time appears to be a main signature thereof. A diffusive character of the emerging patterns has been emphasized, and some elucidation, supporting an understanding of the model phenomenon in question is provided. Excursions towards dynamics of the multiline as well as a relation to known kinetic or dynamic models have been offered as well.

Keywords: multilineal patterns, evolution, discrete space, nonlinearity

1. INTRODUCTION

Attempting to approach an understanding of complicated model phenomena in physics would not have been possible without benefitting from technical possibilities given by computers [1-4]. There exist certain classes of problems in statistical physics, and related branches of physics or chemistry, that by no means can reasonably be treated by even advanced analytical tools. They, accidentaly or not, belong to classes of problems for which no Hamiltonian descriptions have not yet been offered, and which are very suitable for applying a kind of *Monte Carlo* (MC) method, called quite recently a direct MC method [5].

Which are the most leading examples that confirm what has been, perhaps provocatively, stated above? There are quite many. They can be grouped under a common denominator, namely, that they are random in their very nature. It is, however, not a very basic distinction, though can be to some extent. Lucid counterexamples here may undoubtedly be the perennially alive Ising as well as the lattice gas models. They are random but they also have a rather well-known Hamiltonian description both for presence as well as absence of an external field, changing seemingly the overall behavior of each particular system [6]. If they are priviledged to be characterized by such a description, certain dynamics can be assigned to them too. This means that they are dynamic in their nature (clearly, a ferromagnet or a binary melt, or a mixture, are not only kinetic systems but – exaggerting a bit – they are also by no means static.)

There appear, however, such physicochemical systems for which dynamics apparently do not play a decisive role. To state that in another way: There can be certain phenomena for which their dynamics can be much more subtle than we could suspect, *cf*. deterministic chaotic systems, just to obey a Hamiltonian description, or simply some hidden dynamics would be manifested (quantum systems), or certain phenomena arise as nearly equilibrium ones, see percolation, being a model system for studying gelation; random animals, modeling random branched polymers, *etc*. They can mostly be attributed to as kinetic random phenomena. Moreover, they suit very much to computer modeling because they are quite often observed to be of rather discrete, not rarely of Boolean nature, than being defined as continuous systems. One can probably immediately recognize that we have in mind a vast class of physically interesting 'creatures' called fractals, so that we ought to mention right now that another quantitative attribute of them is, that they are well-characterized by their fractal (non-integer) dimension, and are essentially non-Euclidean objects (at least a certain distinguished piece of each of them can have such a property, see the Eden model responsible for description of the tumor or bacterial growth [7]).

To sum up partly, we can state explicitly again that the main attributes of 'computer-made' objects that we are offering to look at are the following: Randomness, kinetic character, discreteness, and finally, fractality; we also should not forget that they are mostly not associated with any Hamiltonian description. To state the same in short:

They are discrete stochastic (kinetic) systems [8], and the physical phenomena from which they emerge are very frequently of nonequilibrium type. Here, as a best chosen example, studied mostly in the eigthies, but also recalled sometimes recently, the diffusion-limited aggregation (DLA) model phenomenon, due to T.A. Witten and L.M. Sander, can be mentioned [9, 10]. (This can serve as a prerequisite of any aggregation or even quasicrystallization phenomona occured on one seed. By the way, to our knowledge no successful Hamiltonian description has until now been offered for DLA.)

Our model belongs to the same "universality" class though it is a little bit more complicated (*viz*., extended) both in type as well as in number of rules as well as in the understanding of them, even with some consequences, inevitably coming from such an understanding. Also, like DLA, the model is not fully understood, but its advantage would be that it is younger, and can rather be compared with surface kinetic phenomena, like kinetic roughening [11], a staff more or less related to the frequently mentioned Kardar-Parisi-Zhang problem, though the latter is anticipated as being dynamic [11]. Some kinetic relation to that famous problem does exist, however, so that we may also have a look at such a loosely appearing connection.

The paper is organized as follows. In Section 2, we present the algorithm and its realizations, certainly not avoiding some technical points to be solved while practically exploring the invented algorithm. In Section 3, we discuss a quite basic feature of our system which appears to be its lognormality both in size (*e.g.* multiline length) as well time (for instance, corresponding to reaching a certain value of the length by the multiline). In Section 4, we would like to reveal the system behavior, commonly termed kinetic roughening. More specifically, we wish to present a formation of some quasicrystals of Eden type, with and without defects. Moreover, in the same section we present a diffusion-controlled formation of emulsion, without going into details related to its electrostatic nature, however. In the next section (Section 5), we make a short (but exemplified) tour towards dynamics of roughening, whereas in Section 6 we collect concluding remarks.

2. ALGORITHM AND ITS REALIZATION

Since in [12, 13] a flow chart of the random multiline evolution has been discussed thoroughly, let us only regard its main points. So, let us start from sketching the algorithm. It can be done as follows:

- Put a starting configuration in a square lattice: Either a straight-line [12] or a rectangular seed [13] are considered in this study;
- Play on a set of random rules, *i.e.* choose a suitable one taken from 8 available rules (3 of reversible type, which means six forth and back rules in total, and some 2 remaining, being of irreversible nature), causing some evolution, or a structural rearrangement, of the multilineal pattern formation under investigation; note that the rules are listed below;
- Go constantly back to the above point unless desired statistics and/or other estimated quantities can be inferred; make sure that all the possible counters involved in the simulational process are updated accordingly.

For introducing the rules in a possibly self-explained way, let us adopt an *ad hoc* algebraic procedure and let us then label four edges (links) of the unit square on which the multiline is assumed to land by *d* (down), *u* (up), *l* (left) and *r* (right), respectively, and assume for simplicity, that *l*-edge (being placed on Y-direction), being just presumed to be occupied, will be shifted rightwards along X- direction, *cf*. Fig. 1 for some exemplified picture of a small system.

Fig. 1. Six snapshots, taken from an evolution of the so-called small system, equivalent to six local successive structural rearrangements of the multiline from a straight line to some nonlinear pattern, according to the set of rules described in Section 2.

Now, one can present schematically 8 principal annihilation-creation (also, neutral) rules, driving the system, just by collecting them in a single formula:

$$X_i^d + X_j^u + Y_k^l + Y_l^r \rightleftharpoons Y_{1-l}^r + Y_{1-k}^l + X_{1-j}^u + X_{1-i}^d ,$$

where: i, j, k, l can take on either 0 (means: empty edge) or 1 (means: occupied edge), and where the following rules, being elementary local rearrangements of the multiline structure are possible to be observed, namely:

- No. 1: The above equation for $i = 1$, $j = k = l = 0$, i.e. with ±2 net gain or loss of the elementary units (creative/annihilative); realize, that the multiline is made of elementary units, called briefly links, and that the length of the link is always equal to the square lattice constant
- No. 2: The above equation for $i = l = 1$, $j = k = 0$, i.e. with 0 net gain or loss of the links (neutral)
- No. 3: The above equation for $i = j = 1$, $k = l = 0$, i.e. with 0 net gain or loss of the links (neutral)
- No. 4: The above equation for $i = j = k = 1$, $l = 0$, i.e. with +2 net loss of the links (annihilative)
- No. 5: The above equation for $i = j = k = l = 1$, i.e. with +4 net loss of the links (strongly annihilative),

where the last two points mentioned above have to be taken irreversibly, so that one has to put " ¸ " into the above basic formula instead of the double arrows indicating reversibility, valid exclusively for the three first points mentioned above.

Notice that the proposed procedure can be thought of to be some extension of a procedure of creation of random paths or surfaces on a digital computer, due to Berg and Foerster [14], exemplified by elementary stochastic processes of both fermionic as well as bosonic nature. A main difference would be, however, that in our model a link is chosen at random by a procedure that is equivalent to sampling a uniform distribution probability, and then by realizing its nearest-neighbor configurational context, in which a structural (local) rearrangement takes place. In the model proposed by Berg and Foerster, in turn, an *à priori* probability, inversely proportional to the length of the multiline just created, is taken into account [14].

As for the boundary conditions (BCs) that are very meaningful in the studies of such a type [12], let us state clearly that we have applied fully periodic BCs. In other words: Our realizations of the above algorithm have utilised a practical implementation of the torus concept, no doubts a concept very natural under such circumstances. Moreover, notice that probably the periodic BCs are generally the most often applied BCs. Many researchers see some advantage while applying periodic BCs in having the results of a simulation as a material likely (easily) comparable with some analytical pretreatment, if this sometimes appears to be the case.

In our experiments as the step of simulation (*SS*) we assume a single iteration of main thread (we use *Win32* platform) which executes one of the above mentioned rules.

It is certainly hard to tell, in which extent it corresponds to the time measured in real physical processes.

Because the proposed model is thought of to be very general, we have had to create an algorithm, which makes possible some easy adaptation to concrete application. We accomplish this task thanks to a suitable coding of the rules and representation of the front in a form of the list of dynamic arrays, so we are able to count not only the entire length of the multiline, but also to control both the number of subfronts or the quasicrystal surface without taking into consideration defects in the bulk. The number of subfronts is rather approximate (it is not possible to pick up exactly this quantity every time). The reason for writing such a statement comes from the following. Namely, it is easy to count all the separation effects, coming into play while applying the strong autoseparative rule No. 3, on the one hand, *cf*. Ref. 12, where that rule was exlusively applied for estimating the number of autoseparation effects of the stochastic front. On the other hand, however, there is a possibly tremendious number of other separation effects, being named weak separation effects, that may come from applying, in principle, all the remaining rules. They undoubtedly conduct the multiline formation process to some states with many patological configurations, *e.g*. certain zigzak-sticking points, *etc*., so that for some well-grown object we can make a certain statistical error while counting all the possible patologies, as clusters of sticking points, induced even somehow by applying the corresponding BCs, which was also mentioned in another study, *cf*. [12], and refs. therein. Perhaps, another (equivalent) answer to the question of the number of subfronts is, that it is a matter of definition what really the separation of sublines means, and whether two suitable sublines are well separated or not, when they have just one or possibly more sticking points with neighboring sublines (note, by the way, that in our model an elementary piece, constituting any subfront, is not a point but a link, rather)? Operation of linking of subfronts (resembling a crossing-over phenomenon, well-known from genetics) has been the most difficult to programming element of the algorithm. In the case of investigation of the full evolution of the front (from initial stage to its disappearance) it is necessary to add the second line. It results from geometry of torus, it is not possible to split it into separated (independent) areas while exploring exclusively the one-line concept. Appart from this our program makes possible to set the probability of realization of the rules. In this way, for example, we can affect Eden cluster growth (number of defects or "length" of a fractal suface). We can also disable partitions of the line (in this mode the program cannot create any "islets" and "lakes").

Fig. 2. Evolution of the multiline taken for 4 successive time steps, measured in the number of simulation steps, $SS = 10^3, 2 \times 10^4, 2.5 \times 10^5, 4 \times 10^6$. (The bigger *SS* the more complicated the resulting pattern can be.)

Finishing this section, we have just to mention briefly that a starting or initial configuration will either be a straight line (with an auxilliary parallel, see above) or a rectangular seed, being thought of to be a quasicrystallization center. Notice that some evolution of the multiline, taken for 4 successive time steps, has been presented in Fig. 2 a-d; here, the initial configuration has been assumed to be a straight line, supported by the auxilliary parallel, *cf.* remarks above.

3. LOGNORMALITY IN SIZE AND TIME

A systematic tendency that undoubtedly arises from our study, manifested both in size (length of the multiline and its square too) as well as in time, observed while inspecting more closely the disappearence time of the multiline [12], can be termed **lognormality**. Lognormality usually means a nearly Gaussian statistical behavior, "Nearly" means here that instead of a linear argument of the corresponding probability distribution one may await a "logarithmic substitution" of that argument. We found such a behavior while thoroughly studying our system. Both its signatures as well as some evidence for that can be anticipated while looking at the pictures given beneath. A quite probably deeply rooted feature of the stochastic system under study becomes a relation that tells us that a stochastic observable, like the multiline length, or its square (some equivalent of the area possibly swept by the line), designated by U / $U(t)$, evaluated in two consecutive time instants t as well as $t + \Delta t$ by means of a corresponding (relative) difference, and related to a time difference Δt, yields

$$\frac{U(t + \Delta t) - U(t)}{U(t)} \propto \varepsilon ,$$

where ε is a random fraction, *cf.* a study on polycrystals and their boundaries [13], being independent of time. (Having an excursion towards nomenclature of the subject under study, let us realize that the above equation is named the law of proportionate effects, or simply a proportionality law. It appears often to be a signature of lognormal behavior in the system, *cf.* [15], and refs. therein.) At this point, it may be clearly ascertained that no formal (*e.g.*, analytical) evidence for such a physical space *vs* time behavior but exclusively a numerical has so far been found, reflecting rather first a visible (say, firm) tendency of the system to reaching such a behavior than to be somehow out of it. This having in mind we wish to conclude what has been stated above about a solid tendency towards lognormality of the discrete stochastic process of multilineal pattern formation that we have investigated.

A numerical confirmation of the tendency towards lognormality in size is shown in a series of pictures. In Fig. 3 a-d such a tendency is manifested for the overall length of the multiline, in Fig. 4 a-d for its square, wheras in Fig. 5 a-d one may look at lognormal fits to the number of subfronts, for 4 consecutive time steps, namely $SS = 10^4, 10^5, 5 \times 10^5, 10^6$, respectively. Let us, by the way, draw our attention to the fact that a mathematical form of our fitting function, $f(x)$, looks like

$$f(x) = \frac{1}{(2\pi)^{1/2}\sigma x} \times \exp\left[\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right].$$

Fig. 3. Tendency towards lognormality in size is shown for the total length of the multiline against the number of simulation steps $SS = 10^4$, 10^5, 5×10^5, 10^6, from top left to bottom right, respectively (the linear lattice size is taken at $L = 200$). The corresponding $(F, <)$ pairs are as follows: 0.077, 6.774; 0.107, 7.399; 0.133, 7.852; 0.147, 8.051, *cf*. the mathematical form of logarithmic Gaussian, given in Section 3.

Fig. 4. Same as in Fig. 3 is shown for the square of the total length of the multiline against the number of simulation steps $SS = 10^4$, 10^5, 5×10^5, 10^6, from top left to bottom right, respectively ($L = 200$). The corresponding $(F, <)$ pairs are as follows: 0.156, 13.551; 0.216, 14.799; 0.267, 15.705; 0.296, 16.103.

Fig. 5. Same as in Fig. 3 is shown for the number of subfronts (sublines) against the number of simulation steps $SS = 10^4$, 10^5, 5×10^5, 10^6, from top left to bottom right, respectively ($L = 200$). The corresponding (F, $\lhd$) pairs are as follows: 0.168, 3.640; 0.148, 4.368; 0.153, 4.840; 0.159, 5.041. (Notice a bit 'unpredictable' or nonmonotonic behavior of F.)

Fig. 6. For the same *SS* numbers four lognormal plots, taken for the variance of the overall length *i.e.*, a direct measure of the roughness of the multilineal pattern, have been shown ($L = 200$). The corresponding (F, $\lhd$) pairs are as follows: 0.171, 2.561; 0.220, 4.213; 0.273, 5.336; 0.297, 5.804.

Fig. 7. Tendency towards lognormality in time is shown for the disappearence time against the number of simulation steps *SS* for six various lattice sizes $L = 8, 12, 16, 20, 24, 32$ from top left to bottom right, respectively. The corresponding six (F, τ) pairs are as follows: 0.890, 6.613; 0.857, 8.252; 0.844, 9.404; 0.840, 10.303; 0.838, 11.036; 0.848, 12.189. (Notice a bit "unpredictable" behavior of F, perhaps due to small lattice or finite-size effect.)

For the same *SS* numbers four lognormal plots, taken for the variance of the overall length *i.e*., a simple measure of the roughness of the line, have been shown in Fig. 6 a-d. Let us underscore that the best lognormal fits have been obtained for the total length squared as well as for the roughness, whereas the worst one is undoubtedly taken for the statistics of the number of subfronts. It is, among many reasons, because of a difficulty of counting that number in the computer experiment.

As for lognormality in time, as was already stated, we have decided to take the disappearence time (a time period in which the random multiline will overlap the auxilliary parallel) [12]. The lognormal behavior is quite readily manifested while looking at Fig. 7 a-f, from the top left to the bottom right, consequently for 6 linear lattice

sizes L = 8, 12, 16, 20, 24, 32, respectively. Interestingly, but a quite commonly anticipated logarithmic Gaussian [13] has been found in some distinctly different study, *i.e*. during a process of optimization of the Internet traffic, namely while studing some characteristics, like the number of downlowded pages in the course of download times, called there a latency distribution of the download times of the INDEX.HTML file on the main page of more than 40 thousand web sites. It was found that the lognormal distribution with its two basic parameters, close to those given in the legend to Fig. 7a, fits the afore mentioned characteristics quite well [16].

4. KINETIC ROUGHENING

Kinetic interfaces, with their fascinating geometrical-temporal behavior and a landmark feature, being termed **roughening**, have been studied mostly on a platform of the nonlinear stochastic partial differential equations (PDEs), often more or less reasonably supported by simulation matters [4, 12]. It is well-known that such two routes of dealing with the nonlinear kinetic problems show up both advantageous directions as well as certain drawbacks due to their essential properties [3, 4]. For example, descriptions based on PDEs are by no means priviledged to look reasonably into a discrete nature of the process in question, whereas even an advanced and complex computer simulation cannot be a good candidate for checking or extending under sufficient control and/or accuracy formal (analytical) descriptions offered in the subject matter.

In the following, we wish to give a patient reader a set of picturesque examples from which it follows that the system under study possesses a clear tendency towards roughening, which can be understood here as a certain, sometimes pretty cumbersome departure (see, pictures) from an otherwise smoothened behavior of the lineal multipatterns under visualization. We wish to underline here that within a suitable range (span) of variables' action, and for a given set of parameter values, one may observe a "critical" power law bahavior, like

$$\langle a \rangle = a_{pf} \langle A \rangle^{\nu_{ch}},$$

where ν_{ch} appears to be some characteristic exponent, whereas $\langle A \rangle$ and $\langle a \rangle$ are some related physical quantities *e.g.*, the (averaged) area $\langle A \rangle$ as well as its "circumventing" averaged radius ($\langle a \rangle$), respectively, the former being implicitly a function of time; note that a_{pf} is a time-independent prefactor (kinetic setpoint), usually of minor importance in standard kinetic descriptions. Most likely manifested kinetic behavior that we have found in our numerical study is a diffusional behavior, so that

$$\nu_{ch} \to \frac{1}{2}$$

can frequently be noticed. (One may thus speak of a diffusional roughening, rather.) It is possible to look at the basis of such a very complex roughening kinetic behavior, emerging in our study while going over the set of pictures, presented in the following subsections.

4.1. Eden-like quasicrystal

By the Eden-like [4, 7] quasicrystal (EQC) we mean an object that has a trivial bulk structure and behavior but reveals a non-trivial surface structure (behavior), especially when compared the former to the latter. Let us realize that we may call the investigated objects crystals since their internal structure is equivalent to the regular structure of the underlying (square) lattice. The prefix "quasi" is attached carefully according to some rather commonly accepted philosophy of the origin of quasicrystalline structure, *cf.* [17].

Ideal quasicrystal: Bubble-like behavior

First, let us announce an evolution of our EQC from a quadratic seed. This is just like a spread in every possible radial directions since, roughly speaking, the EQC has, but on a sufficiently large grid scale, a radial symmetry, *cf.* Fig. 8 a-b for a possible realization. A final structure resembles a quasi-circle with stochastically perturbed perimeter line, and clearly looks like an Eden object, no matter which rule was applied to create it [4, 7].

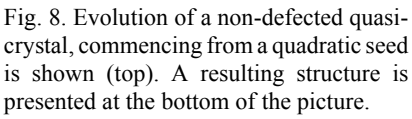

Fig. 8. Evolution of a non-defected quasi-crystal, commencing from a quadratic seed is shown (top). A resulting structure is presented at the bottom of the picture.

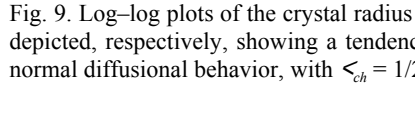

Fig. 9. Log–log plots of the crystal radius (top) and roughness (bottom) *vs SS* are depicted, respectively, showing a tendency of the evolving system to arrive at a normal diffusional behavior, with $<_{ch} = 1/2$.

No defects can be observed inside the object's body, so that EQC resembles a bubble expansion rather than a crystalline evolution. It also reveals, to a large extent, a typical power-law behavior, with the characteristic exponent $<$. 1/2, see Fig. 9 a-b for the corresponding log–log plots for the radius *vs SS* as well as the roughness *vs SS* dependences, respectively. Second, let us note that both the radius of the EQC as well as its roughness do obey the same power (diffusional) law, so that it seems to be no essential differentiation between the bulk as well as surface characteristics, given in a certain range of parameter values. It should be realized that these facts have been reported elsewhere [4, 3, 1] but we do propose here a different method and algorithm to confirm it.

"Real" quasicrystal: Playing on rules means creating defects

A tour towards reality would be if we would be able to create defects in the internal EQC structure. This is also possible while playing patiently on the rules [18] being at our disposal, *cf.* Section 2. Two examples chosen,

where massive (point-like) as well as selective (linear) defects' creations are demonstrated, can be seen in Fig. 10 a-b. In the former, the EQC is built up by readily exploring rules No. 1 and No. 4, and some allowance of many subfronts is to be anticipated here, while in the latter all the so-called constructive rules, leading to a net enlargenment of the object (being of compact character; moreover, one highly defected line is seen, and some blockage of separative rules is realized), have been engaged.

Fig. 10. Structures of defected quasicrystals, with point-like ("vacancies"; "interstitials") as well as linear ("dislocations") defects, from left to right, respectively.

4.2. Diffusion-controlled emulsion formation

Dispersions, like colloids, are always challenging task for almost every study. Among them emulsions or small liquid drops, *e.g.* fat globules in an aqueous medium of digestive tract, appear to be still intriguing as well as a bit misterious, they are also quite well understood [19].

Within the framework of our modeling we have also tried to simulate in a simplified manner some emulsion formation, mostly by starting from a rectangular seed and playing mainly on the autosepara tive rule No. 3, vorbidding however the last strongly annihilative rule. In a more detail, such a play relies on enabling the rule No. 3 always in the case, in which some partition of a big drop into smaller ones does occur. Otherwise, it is not enabled. In other words, a re-merging of drops is vorbidden this way.

Fig. 11. Computer-made emulsion (a mature growing stage; $L = 200$, initial square configuration 50×50) is drawn on top of the picture, according to the set of rules listed in Section 2, with absence of the rule No. 5, but with extensive use of rules No. 3 and 4, promoting a dispersion effect (the account for dispersed phase is like 1/16). A broad range of some clear manifestation of the one-half exponent diffusional behavior is given at the bottom of the plot in a log–log scale, using a bigger lattice size $L = 250$.

The purpose of such a study can be understood as a kind

of modeling of the formation of prerequisite of some (micro)emulsion, being thought of as a process of successive but accidental division of a big rectangular seed, which is separated into parts in a random manner. (It is suspected, that such a mechanism will be of use in systems, in which a role of capillarity effects is negligible, and where the influence of potentials certainly involved in the system evolution, like *DLVO*, keeps its diffusive nature survived; perhaps, it is possible while metabolizing fats by emulsifying them, because such a process goes preferentially by chance.) As a result, you may look at what is demonstrated below. Let us state additionally, that the gross simplifications we speak about are mostly due to total neglect of electrostatics associated with each emulsion formation process, and we do not take into account a possible dependence upon concentration of dispersed species [18]. The observed behavior reveals clearly its diffusional character, *cf*. Fig. 11 a-b, though some discussion can be taken up about which is the role of surface tension in the described model process [18].

5. TOWARDS DYNAMICS OF ROUGHENING OR HOW TO PROCEED FURTHER

Models that manifest, however, their dynamic character, and that can be "in the same basket" as our model, are mostly those having much in common with Ising ferromagnet, and mostly with the motion of domain walls in the domain ferromagnetic structure [20, 21]. They exploit the conception of self-avoiding random walk as a real substitute of the perimeter structure. They are, however, based on the rules No. 1 and No. 2 (see above) though they assume – like in a typical Monte Carlo – that an increase of the total length of the multiline can be realized with a probabilistic Boltzmann weigth, P, like [20]

$$P = \exp(-2\Sigma\beta),$$

where Σ stands for the line tension (see, our rule No. 3) and $\beta = 1/k_BT$ has the meaning of the inverse thermal energy of the system, being more often termed the inverse temperature. Notice that the above systems are Hamiltonian systems, *i.e*. the Hamiltionian description is known for them [20, 21], and numerical investigations locate the systems to be in a class of self-avoiding walk, a concept the polymer specialists are very familiar with [22]. To conclude, we may state that while embarking readily on dynamics we ought to consider a repeated overall experiment in which all possible runs will reasonably be realized with the probabilistic weigth given above; this is like to go from a direct MC to a Metropolis-like MC within the described subject matter.

CONCLUDING REMARKS

Concluding remarks, based on what has been discussed before, can be listed as follows:

- The multilineal evolution under study appears to be a complex kinetic process, in which irreversibility as well as prevailing annihilative character in the course of the simulation time do win eventually against reversibility and creation, so that a possible application of the proposed modeling would be to study a dissolution of the diffusion-reaction front.
- The description presented is a purely simulational (numerical) approach the striking signatures of which are mostly lognormality and diffusional character, at least within the framework discussed in this study.
- The model offered seems to be very suitable for studying a roughening process controlled by diffusion, or even, the so-called roughening transition [4].
- An extension, incorporating dynamics seems to be plausible, *cf.* the preceding section, and is going to explore some interplay between the free line and thermal energies, see also [23] in this respect.
- A vast range of applications of the proposed modeling has been given in [12], and to a certain extent in [18], and can even be extended, *e.g*. while looking carefully into [24].
- Technical details, concerning the presented modeling, have mostly been offered in [12], and refs. therein, and also in [18, 23, 13].

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References

[1] H. J. Herrmann, Phys. Rep. 136 (1986) 153; T. Vicsek, *Fractal Growth Phenomena*, World Scientific, Singapore, 1992.
[2] F. Family, T. Vicsek, eds., *Dynamics of Fractal Surface*, World Scientific, Singapore, 1991.
[3] F. Family, D. P. Landau, eds., *Kinetics of Aggregation and Gelation*, North Holland, Amsterdam, 1984.
[4] T. Halpin-Healy, Y.-C. Zhang, Phys. Rep. 254 (1995) 215.
[5] D. W. Heermann, *Podstawy Symulacji Komputerowych w Fizyce*, WNT, Warszawa, 1997.
[6] J. J. Binney, N. J. Dowrick, A. J. Fisher, M. E. J. Newman, *Zjawiska Krytyczne*. *Wstęp do teorii grupy renormalizacyjnej*, WN-PWN, Warszawa, 1998.
[7] M. Eden, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, edited by J. Neyman, University of California Press, Berkeley, 1961.
[8] L. Schimansky-Geier, T. Pöschel, eds., *Stochastic Dynamics*, Springer-Verlag, Berlin, 1997.
[9] T. A. Witten, Jr., L. M. Sander, Phys. Rev. Lett. 47 (1981) 1400.
[10] C. Moore, Material presented at the NATO ASI "Complexity from Microscopic to Macroscopic Scales: Coherence and Large Deviations", 17-27 April 2001, Geilo, Norway.
[11] H. Gould, F. Family, H. E. Stanley, Phys. Rev. Lett. 50 (1983) 689; A. Christou, R. B. Stinchcombe, J. Phys. A: Math. Gen. 19 (1986) L454.
[12] A. Gadomski, M. Schönhof, K. Bończak, in *Stochastic Processes in Physics, Chemistry and Biology*, edited by J. A. Freund and T. Pöschel, Lecture Notes in Physics No. 557 (Springer–Verlag, Berlin, 2000), pp. 496-506.
[13] A. Gadomski, Fractals, Vol. 9 (2001) in press.
[14] B. Berg, D. Foerster, Phys. Lett. 106B (1981) 323.
[15] M. Niemiec, A. Gadomski, J. Łuczka, Acta Phys. Pol. B 32 (2001) 1513.
[16] D. Helbing et al., in *Traffic and Granular Flow'99*, edited by D. Helbing, H. J. Herrmann, M. Schreckenberg, D. E. Wolf (Springer–Verlag, Berlin, 2000), pp. 193-204.
[17] D. Levine, P. J. Steinhard, Phys. Rev. B 34 (1986) 596.
[18] M. Cyrankiewicz, *Diploma Thesis*, University of Technology and Agriculture, Bydgoszcz, 2001.
[19] J. Lyklema, *Fundamentals of Interface and Colloid Science*. Vol. I, Academic Press, London, 1991.
[20] D. Stauffer, D. P. Landau, Phys. Rev. B 39 (1989) 9650; D. Stauffer, N. Jan, Canad. J. Phys. 66 (1988) 187; R. H. Swendsen, Phys. Rev. B 15 (1977) 689.
[21] M. Cieplak, M. Suan Li, Fractals 2 (1994) 481.
[22] W. Przygocki, A. Włochowicz, *Fizyka Polimerów*, WN-PWN, Warszawa, 2000, chaps 1-4.
[23] M. Schönhof, *Diploma Thesis*, Humboldt University at Berlin, Berlin, 2001, chap. 4.
[24] A. A. Adamson, *Physical Chemistry of Surfaces*, Interscience Publishers, Inc., New York, 1960.