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Comment

Comment on "How skew distributions emerge in evolving systems" by Choi M. Y. et al.

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Emergence of skew distributions such as those of power law, log-normal, or Weibull types is ubiquitous in both nature and man-made systems [1,2]. However, it still lacks proper and possibly unifying, also general, understanding of the governing mechanism and relations between the distributions mentioned [3].

In [1] it was studied how such distributions emerge in general evolving systems and what makes the difference between them. The authors began with a master equation for general evolving systems, obtaining then the time evolution equation for the size distribution function, $f(x,t)$ [1], where x is the object's size (t, evolution time). They did confine their derivation to the case of size changes of a mesoscopic constituent (a grain size in our case, cf. [2,3]) proportional to the current (object/grain) size. They derived the distributions of a power law type in stationary conditions [1,4], $\partial f(x,t)/\partial t = 0$, and when escaping from them either log-normal or Weibull-type distributions emerge asymptotically, depending on production and growth in the system. Moreover, they claim that this master equation approach may give a unified description of those three types of skew distribution, at least as far as their generalized forward-Kolmogorov–like scheme (eq. (10)) is concerned [1].

In a series of papers on grains-containing and diffusion-type (underlying Fokker-Planck–type dynamics in x-space) evolving systems [2–5], depending on the boundary conditions applied, we have also derived rigorously either algebraic distributions [4] or several types of Weibull distributions, irrespective of whether the system evolves in constant total-volume conditions or escapes from them [2,3]; the derivation is, however, not completely equivalent to that of [1]. The only general requirements for the system to evolve in a d-dimensional Euclidean space ($d = 1, 2, 3, \ldots$) were, in case of $d > 1$, existing possibilities of emergence of local "unruly" surfaces (grain boundaries), in a $(d-1)$-dimensional evolution subspace, cf. [2,3], indicative of the grains' presence in the system; this very presence of them has mainly been embodied in the Fokker-Planck (or, equivalently, forward-Kolmogorov) type entropy-involving [5] dynamics by means of the surface-dependent diffusion coefficient, $D(x) \propto x^{(d-1)/d}$, like $\partial f(x,t)/\partial t = D(x)\partial^2 f(x,t)/\partial x^2 + D(x)\beta(\mathrm{d}\Phi(x)/\mathrm{d}x)\partial f(x,t)/\partial x$, with β, an inverse thermal energy [5], and $\Phi(x)$ the free x-dependent system's energy [2,3]. To arrange the proper grainy matter distribution in d-dimensional space, clear signatures of random close-packing, quantified by a Kolmogorov phase-change[1] index, $\kappa(d) = d+1$, emerged, the latter being characteristic of atomic or molecular elementary arrangements in polycrystalline or partly amorphous systems [4,5]. The modeling distinguished then whether the role of the surface is mainly to provide the surface tension effect, wherein an involvement of curvature(s) appears to be decisive [3,4], or when focusing on the case that the surface, obeying its catalizing task, provides the first-successful-attachment physical cause, as is in the colloid-type, more high-temperature or on-entropy-impact aggregations [2,5].

Since the authors of [1] claim that their derivation may possess a unifying character, and suits certain general albeit linear evolution rules, very characteristic of the log-normally distributed random events [3], as is expected to occur in the Yule process [1], we wished to argue in clear terms quite the opposite, namely, that

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[1]Both self-organized *viz* (mesoscopic) nonequilibrium-thermodynamics [5] grainy evolutions discussed follow the (nucleation-growth) phase-change rate as $\mathrm{d}\langle R(t)\rangle/\mathrm{d}t \sim t^{-\nu_{phc}}$, $\nu_{phc} = d/\kappa(d)$, with $\langle R(t)\rangle$ the average grain's radius [2,3] taken at time t.

neither a unifying character nor some considerable impact of generality stays behind the proposed derivation of the (non)stationary distribution(s) [1]. Of course, the rigorous fingerprint of the derivation goes without saying, as looks also the concise and informative enough presentation of the derivation scheme, and its scholarly usefulness [1].

There is one working assumption, legitimate from the mathematical point of view that can, however, be questioned as too oversimplified physically, especially when confronting it to the quite general message sent by the letter [1]. This is the exponential law, placed in [1], cf. eq. (4) and below it, being stated in an ordinary-differential form, such as $\mathrm{d}N/\mathrm{d}t = rN^{\chi}$, where N is the number of elements the evolving system is equipped with, r, the production rate (being virtually related to the entropy production rate in grains-containing systems, prone to self-organization, cf. [5]). The only exponent $\chi = 1$ is proposed by the authors of ref. [1] —this ensures the overall derivation of the skew distributions to go smoothly, cf. the passage between eq. (4) and (5) in [1]. This, in our opinion, limits very much both the unifying character and generality of the mechanism they are able to offer for even standard physical situations to be expected in evolving polycrystalline materials [2–5]. For them the (average[2]) number of constituents (grains), taken in the long-time regime, also conforms to $\mathrm{d}N/\mathrm{d}t = rN^{\chi}$ but with $\chi \equiv \chi(d)$, wherein $\chi(d) > 2$, thus exceeding, even distinctly the value $\chi = 1$. A careful analysis of nonconstant- [2] and constant-volume [3] material evolutions, respectively, shows that it is really the case. It points via $\kappa(d) = d + 1$ to $\chi(d) = 1 + [\omega(d)]^{-1}$, with some $\omega(d)$ of either $\omega(d) = 1/\kappa(d)$ or $\omega(d) = d/\kappa(d)$, yielding then $\chi(d) = 1 + \kappa(d)$ or $\chi(d) = 1 + [\kappa(d)/d]$ for these two types of diffusion-type evolutions [2,3], respectively, this way unraveling them as d-dependent (also, temperature-dependent) processes[5]. Finally, a power law (in contrast to an exponential $N(t) \sim \exp(rt)$ in [1]) emerges, namely, $N(t) = Ct^{-\omega(d)}$ results, with $C > 0$, a constant, pointing to a number of grains in a starting configuration. (Notice that $\omega(d)$ takes on fractional values for the constant-volume evolutions studied [3] which is, however, not the case of the second counter-evolutions, with negligible role of surface tension of mesoscopic constituents [2]. For the former the respective speeds of the evolution measured in terms of both $N(t)$ and $\mathrm{d}\langle R(t)\rangle/\mathrm{d}t$ (phase-change rate) coincide completely in their temporal behaviors —from this perspective, the self-organized evolution governed more by surface tension changes looks more optimal [3,4].)

Thus, the exponential assumption proposed in [1], cf. eq. (4) and below it, looks fairly legitimate on trial but does not follow entirely the emerging physical peculiarities, at least the ones attributed to the grains-containing evolutions [2–5]. Especially, since such evolutions may also follow nanoscale fractionalized principles [5], there appears a need to state their kinetics not in the first-order (Avrami-Kolmogorov [3]) crude way but to redefine them in appropriate terms of typically fractional *viz* anomalous kinetics, with some exponent $\chi(d) > 2$. Note that now the grainy-matter rate $r = r(d)$ reads $r(d) = 1/\mu(d) \times C^{-\mu(d)}$, with $\mu(d) = 1 - \chi(d) < 0$, which suits well the overall dimensionality-dependent character, cf. [2,3,5] for details.

Of course, in the case of the surface-involving, jammed and fluctuating systems considered [4,5], the only exclusion of surface-less (thus, normal-diffusion) involvement emerges naturally at $d = 1$. In this case, the size distribution of grains is normal, see [1–3], contrary to all cases of $d > 1$ that are nonstationary skew-distributed, mainly in a Weibull-type manner [2,3], or follow a power law (with an exponent $0 < 1/d \leqslant 1$) in a certain stationary, prone-to-phase-separation regime [4]. Explicit solutions of Weibull-type distributions derived for condensed matter (polycrystalline) systems, analogous to those presented by us for $d > 1$ [2,3], can also be found in [6]. The case of $d = 1$ has been presented in [7] as a diffusion-type model based on Fick's 2nd law in the space of grain sizes, whereas its Langevin-type counterpart, referring to random motion of grain boundaries one may find in [8]; for an analytical approach to the normal grain (or, model lipid domain) growth of d-dimensional (bio)materials one is encouraged to look into ref. [12] of [2].

Summing up, the generally useful derivation scheme of $\mathrm{d}N/\mathrm{d}t = rN^{\chi}$, presented for $\chi = 1$ [1], can show up its only fairly limited robustness, exemplified at most by the unconfined self-organized phenomena of economical or sociodynamic, also bio-populational, and exponential character [1]. It cannot be applied to the more confined, logistic (or, Verhulst-type) and entropy production [5] systems, such as evolving metallic or ceramic polycrystals and soft mesomorphs with grain-boundary barriers [8], undergoing the Fokker-Planck (forward-Kolmogorov [1], cf. eq. (10) for $n = 2$) type grain-surface and d-dependent dynamics [2,3] —for them, the requirement of $\chi \neq 1$, with $N(t) \sim t^{1/1-\chi}$ (to compare with formula (50) from [3]), manifests solely. It is also because if $\chi \neq 1$ were put in formalism [1], its production rate $r \to 0$ at stationarity, thus yielding x-independent distribution *viz* eq. (9) of [1] with $\alpha = 0$, pointing again to the limited applicability of [1].

[2] Evaluated in terms of the *0th* central moment of the stochastic process, namely $\int_0^{\infty} f(x,t)\mathrm{d}x$ [2,3].

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